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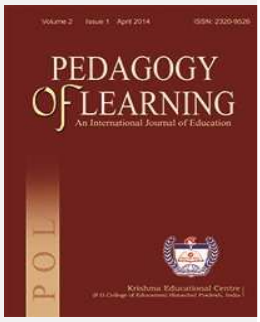

Redesigning the Mathematics Classroom through TPACK enriched Pedagogical Strategies

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Abstract
Technology continues to change with the changing demands of society. With the integration of new technologies in the classroom, the evolution of changes in classroom instruction has been a perpetual feature. But, the efficiencies of teachers vary from each other considering their way of integration of content, pedagogy and technology with each other. For effective teaching-learning process, a balanced knowledge of these three components is expected on the part of teacher i.e. an adequate level of Technological Pedagogical and Content Knowledge (TPACK). The conventional pedagogies need to be strengthened through the application of technology or can be replaced if situation demands for so. As such the application of flipped teaching improves the learning of students. Mathematics is considered as least preferred subject so new pedagogies should be thought of for better comprehension and achievement of students. Innovative pedagogical strategies like animated content demonstration may support in increasing the engagement of students in mathematics. The dynamic and interactive learning environment can be created in the classrooms through the interactive software like GeoGebra, Geometer Sketchpad and the activities based on concrete and virtual manipulative can contribute in enhancing the effectiveness of technology oriented pedagogies.

Keywords: TPACK, Mathematics, Pedagogical Strategies, Technology

1. Introduction

Mathematics is observed as a powerful social entity as it helps an individual in dealing with various spheres of life i.e. private, social and civil. It serves as a catalyst in the economy of a nation as the mathematical competencies are required in almost all fields like politics, corporate sector and social planning. Moreover, according to some latest space related researches, the communication with the aliens should also be done by using mathematics (David, 2017). Thus, mathematics is a discipline as well as an interdisciplinary language which is considered as one of the 'basics' in the formal system of education.It is the science of numbers, quantities, shapes and their relationships. It involves the operations like addition, subtraction, multiplication and division. The branches of mathematics are namely, algebra, arithmetic, geometry, calculus and trigonometry. Mathematics helps in developing certain qualities in an individual like logical reasoning, critical thinking, spatial thinking, problem solving ability and creativity.

2. Need of change in Pedagogical Strategies for Mathematics

Now days, while studying mathematics, students are confronted with some challenges like memorizing the formulas, tables and losing interest as a result of being unable to relate the topics of mathematics to their real life. Because of such challenges, they either drop-out or fail to perform better in the subjectand begin to consider mathematics as a tough subject. Mathematics teacher can play an important role in better engagement of students while learning mathematics. The success of mathematics teaching lies in the way the teacher considers the core dimensions of teaching.

The traditional pedagogical strategies like inductive-deductive method, analytic-synthetic method, problem solving method, laboratory method, play way method and teaching through pedagogic resources (Charts, manipulative, programmed learning materials and activities) are used by the teachers while teaching mathematics.

As the time passes, the change occurs in the needs, interests of students and the environment surrounding the students. This change is mainly related to the innovations resulting from the developments in science and technology. The selection and application of various methods of teaching need to be in correspondence with the content, level of students and the changes occurring in the present scenario whether technological or social.Thus, the teachers need to think of the new ways for the presentation of material to the students to make it more understandable and interesting for them. They need to have the knowledge about the activities or methods which are suitable for the particular content for enriching the classroom atmosphere and for gaining attention of students.

Considering the digitalization around the world and the interest of students in gadgets or other forms of technology, the pedagogical strategies can be replaced or modified by giving the digital touch to the traditional pedagogical approaches wherever needed. According the Technology Principle of National Council of Teachers of Mathematics, the technologies are essential tools for the balanced mathematics program and teachers must serve as knowledgeable decision makers in determining the effective utilization of technological tools by the students i.e. the teacher must be able to decide when and how the technological tool will lead towards effective learning of students (NCTM, 2007). This can

be possible if the teachers have an adequate level of Technological Pedagogical and Content Knowledge i.e. TPACK.

3. TPACK and Innovative Pedagogical Strategies

TPACK framework was given by Mishra& Koehler in 2006 which is an extension of PCK (Pedagogical Content Knowledge) articulated by Shulman in 1986. Originally TPCK was the acronym given to the framework which was latermodified to TPACK for easy pronunciation (Chai, Koh & Tsai, 2016).The TPACK framework involves the equal application of TK (Technological Knowledge), PK (Pedagogical Knowledge) and CK (Content Knowledge). TPACK is significantly and positively correlated with its elementary componentsTK, PK& CK (Chai, Koh & Tsai; 2010).

These elementary forms of knowledge i.e. TK, PK and CK on further intersection result in secondary forms of knowledge i.e. TCK (Technological Content Knowledge), PCK (Pedagogical Content Knowledge) and TPK (Technological Pedagogical Knowledge).

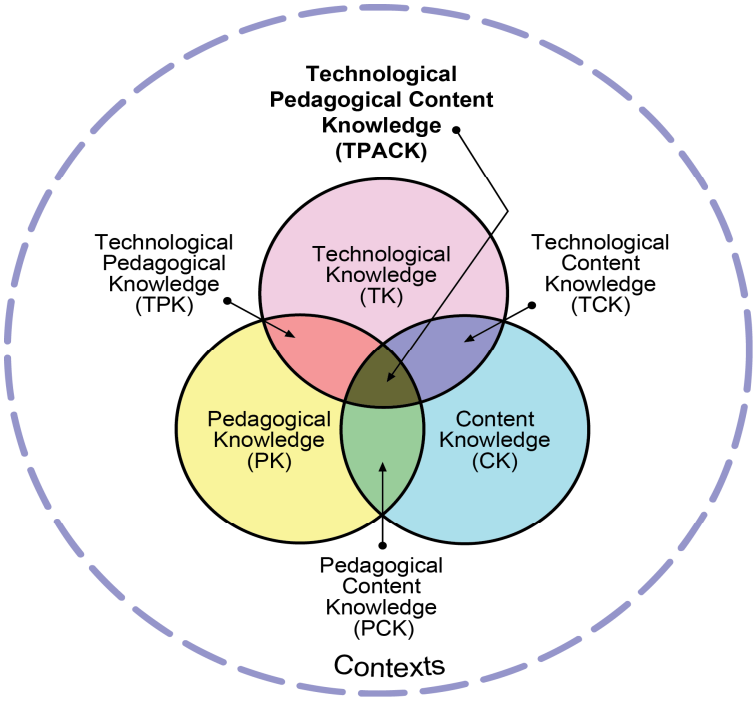

Figure 1: TPACK framework (source: www.tpack.org)

For integrating technology in the classroom mathematics teaching, a teacher is required to have an adequate level of Technological Pedagogical and Content Knowledge. TPACK will help the teacher in appropriate selection and integration of technology in the pedagogy to be used and the content to be taught to the students. The vision of TPACK is also supported by the National Council of Teachers of Mathematics (NCTM) which emphasizes on the importance of technology by stating that technology has an impact on the teaching and learning of Mathematics subject. It helps in the improvement of students' learning (NCTM, 2000). It is also emphasized by NCTM that the teachers need to be prepared for meeting these standards. In order to create such environment for students, the

teachers must also experience the technology incorporated environment. (NCTM, 2007). TPACK in mathematics education is the interplay of

- Teachers' knowledge about the mathematics' content
- Methods of teaching mathematics that are known to the teacher
- Knowledge about technologies that can be appropriately integrated in mathematics classroom for the better understanding of mathematical concepts'.

To rethink about the pedagogical strategies for mathematics in the era of digitalization, each dimension of TPACK will play an important role. A major role will be played by TK and PK in deciding the innovative pedagogies for mathematics. Similarly, the intersection of TK and PK i.e. Technological Pedagogical Knowledge (TPK) will have the major contribution among all the secondary forms of knowledge under TPACK for using innovative pedagogies in the mathematics classroom. Many traditional methods of teaching are used by the mathematics teachers to teach mathematics like lecture cum demonstration method, inductive-deductive method, analytic-synthetic method etc.

3.1 Demonstration of Concrete & Virtual Manipulative

The use of manipulative in mathematics education has been recommended by the educators for a long time (NCTM 1989, p. 17). In the traditional classrooms, while teaching through lecture cum demonstration method,the teacher used to teach the topics like fractions, measurement, data analysis and geometry etc. with the help of concrete or physical manipulative. Concrete manipulative are the real objects like blocks, coin, geo board, chalk and other real objects which are used to give demonstration while giving lecture to the students. One may miss the concept while learning with the help of them as numbers cannot be seen on them. Also, there is no provision of feedback in case of learning through these concrete or real manipulative. Considering the topic fraction, the concrete manipulative do not allow to add or subtract fractions mentally.

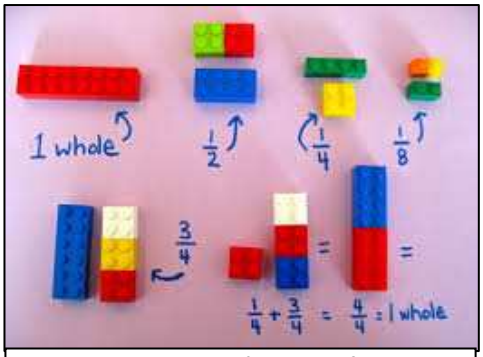

Figure 2: Learning 'Fraction' through concrete manipulatives (Source:http://www.mathnasium.ca/2015/12/legos-an-excellent-math-manipulative)

In this 21st century, virtual manipulatives are taking place of these traditional concrete ones. Virtual manipulatives are the replica of the real manipulative which can be accessed through internet. It is interactive, web based representation of dynamic objects. It gives opportunity to students to construct their knowledge as result of their actions taken by clicking or dragging the dynamic virtual objects. Instead of just seeing on the computer screen, the learner can engage more by interacting with the dynamic objects.

There are many virtual manipulative applets like NLVM Fractions Adding, NCTM Illuminations Fractions Model I, NLVM Base Blocks addition and NLVM Sieve of Eratosthenes which the teachers can use while teaching mathematics at elementary level. NLVM Fractions Adding tool involves the addition of fractions with unlike denominators. The students work on finding common denominators and making connections between the symbolic i.e. numerical and the pictorial or fraction models. The Illuminations Fraction Model I involves the activity of making connections among the fractions, decimals and percent. These connections are made by sliding the numerators and denominators. In the NLVM Base Blocks Addition Tool, the students can explore their problems related to addition of base-ten numbers by using the base-ten blocks. Thus, the students compose or decompose the numbers by moving the blocks across tens places.

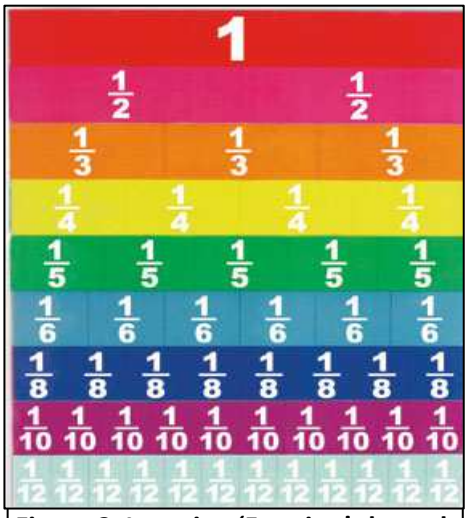

Figure 3 Learning 'Fraction' through virtual manipulatives
(Source: http://www.si-manufacturing.com)

The NLVM Sieve of Eratosthenes Tool helps in identifying the prime and composite numbers between 2 and 200. If the student clicks on a number 4, then all the multiples of 4 are automatically removed and by continuing like this, the students identify the prime numbers left behind. The mathematics teacher can integrate such technology tools for stimulating interest in them while teaching the concepts of mathematics.

Both virtual and concrete manipulatives result in higher achievement and both increase the flexibility in representation of algebraic concepts (Suh & Moyer, 2007). Virtual manipulatives are found to be beneficial for the students with learning disabilities. They provide flexible options for learning and leads to autonomy of students. In addition to it, the virtual manipulatives equip the teachers with wide range of options to facilitate the students with special and diverse needs (Satsangi & Miller, 2017).

Among various disorders, autism spectrum disorder is one of the disorder,suffering from which, the students requirevariety of activities and special attention on the part of teachers. Such students may also be benefitted with the use of virtual manipulatives in the mathematics learning. While teaching the subtraction skills to elementary students with autism spectrum disorder, the students who use virtual manipulatives arefound to be more accurate and independent in solving the problems of subtraction in comparison to the students who use concrete manipulatives (Bouck, Satsangi, Doughty & Courtney, 2014).

Although, the virtual manipulatives are seen as an effective replica of real or concrete manipulatives, yet there is a need to see the suitability of these manipulatives according to the resources available with the teacher and the content to be taught which is related to the issue of PCK and TPACK, as a whole. A teacher can also employ the virtual and concrete manipulatives interchangeably or in progression from concrete to abstract ones as the virtual manipulatives do not give hands on experience to the students. Thus, the choice and employability of manipulatives will ultimately be based on teacher's TPACK.

3.2 Animated Content Demonstration

Animated GIFs (Graphics Interchange Format) are also in trend for teaching various topics of mathematics. These animations facilitate in the learning of mathematical concepts which involve some working like Pythagorean Theorem demonstration which is more understandable by showing the rotation of three squares. Such working is difficult to demonstrate by the teacher in real sense as it will require a lot of time and energy to prepare such models. In such cases, the teacher can use the animated demonstrations in the mathematics classroom. The transition from concrete to abstract concepts is the most important step in mathematics learning and animations can fully serve this purpose. Animated texts are found to facilitate the cognitive processes of students which enables them to be specialized in selecting information, constructing representation models and promoting their learning ability of basic mathematics concepts (Luzon & Leton, 2015).Just as Pythagorean Theorem, many other topics like surface area of cylinder, real world applications of Sine, Cosine and Tangent etc. can be best demonstrated through animated GIFs. Thus, the teacher with an adequate technological pedagogical content knowledge can use such animated GIFs for the better understanding of mathematical content by the students.

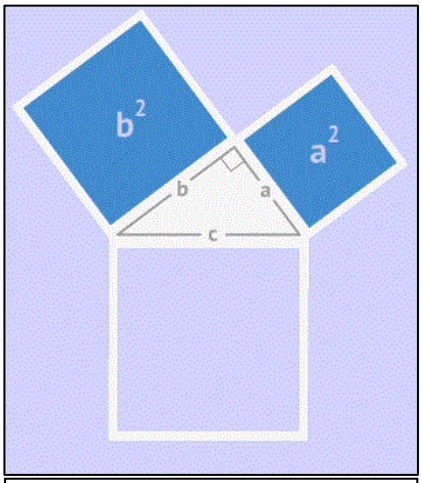

Figure 4 Animated GIF of Pythagorean Theorem (Source: www.mathwarehouse.com/gifs)

3.3 Interactive Learning through Dynamic Mathematics Software

The mathematics teachers can create the dynamic and interactive mathematics learning environments (DIMLE) for extending the mathematical knowledge and understanding of their students. As majority of students consider mathematics as a boring subject, the teacher can create an interactive learning environment through various interactive software for mathematics GeoGebra (allows to make constructions with points, lines, vectors, inequalities, polynomials etc. and to change those constructions dynamically), Fathom (a dynamic software for teaching data analysis, statistics, algebra and pre-calculus) and Geometer Sketchpad(a number of objects can be created, measured and used for solving mathematical problems).

Dynamic Mathematics Software Geo Gebra increases the attention and motivation of students by providing the visual and concrete learning environment to them resulting in their better conceptual understanding and achievement (Tatar & Zengin, 2016). The Fathom Dynamic Statistics software provides opportunities to the students to drag and drop the variables into the graph which helps them in visualizing the effect of change in data on the related measures. This feature of dragging and dropping makes the software more user friendly for the students(Agudo, Sanchez, & Rico, 2010).

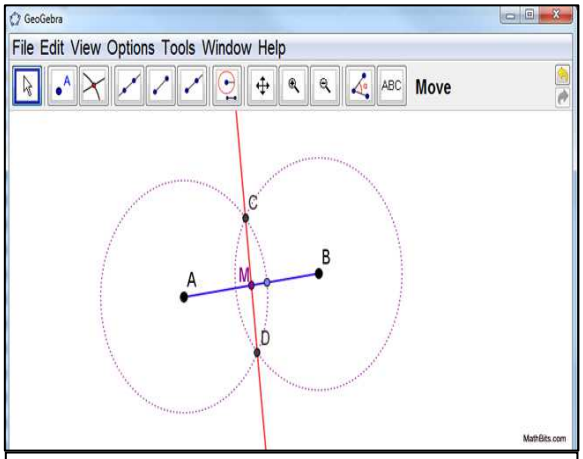

Figure 6: Construction of a segment bisector using GeoGebra software

(Source:https://mathbitsnotebook.com/Geometry/Constructions/CCSortware.html)

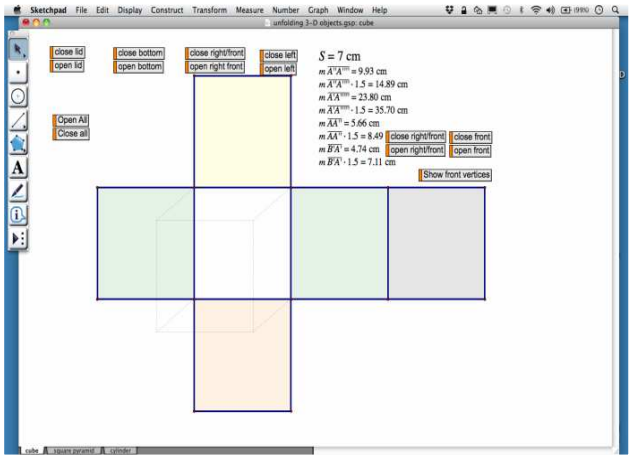

Figure 5: Unfolding the 3-D objects using Geometer sketchpad

(Source:http://teachingisagift.blogspot.com/2014/12/loved-that-lesson-technology-edition.html)

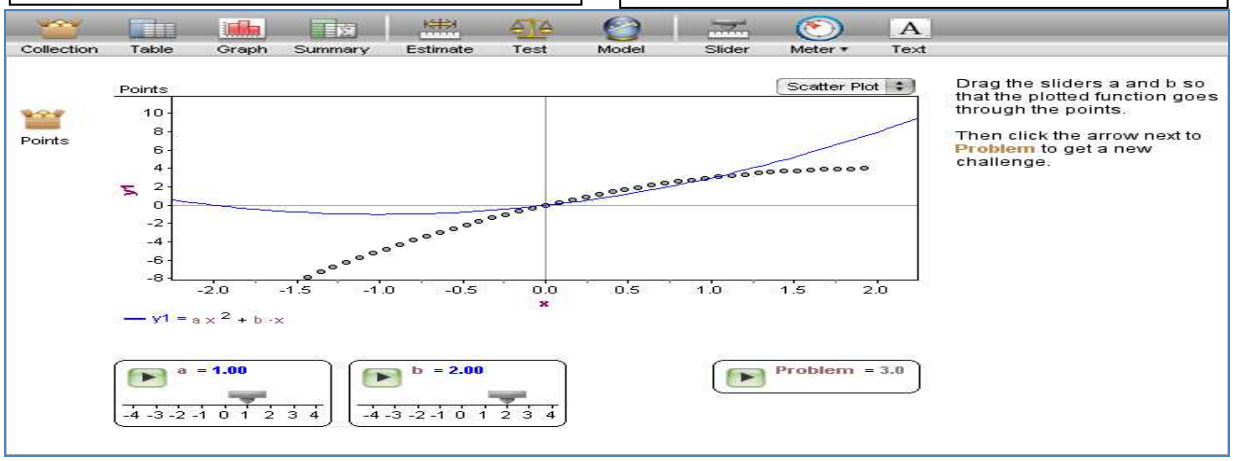

Figure 7 Plotting of values and functions using Fathom
(Source: https://fathom.concord.org/)

Also, the dynamic mathematics software (technology), when supported with the cooperative learning model (pedagogy), results in increased understanding and achievement of students in the topics 'quadratic functions' and 'sequences' (content) (Zengin & Tatar, 2017). Using Interactive whiteboards with the dynamic software enables the students to have an in-depth understanding of the concepts (Lavicza & Papp-Varga, 2010). The interactive dynamic software for geometry include a range of activities for deeper reflection and more advanced exploration in comparison to the geometry learned through paper and pencil (Straesser, 2001). It clearly indicates that the mathematics teacher should have an updated knowledge about integration of such dynamic software while teaching mathematical concepts.

3.4 Flipped Classroom Approach

In flipped teaching, alterations of two activities i.e. lecture and homework takes place for active involvement of students in the teaching-learning process. Lectures are delivered at home and homework is done in the classroom, thereby, flipping the classroom. In addition to the swapping of locations, flipped classroom approach also involves the replacement of classes with the videos or any other activity which can be done by the students on their own at home and the classroom involves the discussion and the practical activities carried out by the students cooperatively. Thus, considering the Bloom's taxonomy of instructional objectives, knowledge and comprehension is achieved at home and rest of the objectives in the hierarchy is achieved in the classroom.

Flipped classroom approach provides visualization to the abstract concepts of mathematics. When used with Khan Academy and mathematics software, this flipped classroom approach promotes better retention, understanding and achievement of students (Zengin, 2017). The flipped classroom approach requires the active learning of the students. The conventional lecture method in flipped classroom approach can never serve the purpose and makes it ineffective. Thus, the mathematics teacher should try to optimize the intellectual engagement of the students by adding some activities like asking questions in the videos of mathematics to enhance their learning as well as to assess their understanding of concepts explained in the videos. The questions can be multiple choice, true/false or open ended. The multiple choice questions can serve the purpose better as the students can be graded immediately using the software programs and also they are less time consuming for students. In line with the questions embedded in the videos, the mathematics teacher can add other pedagogical techniques for the better engagement of students in flipped classroom approach.

To flip the classroom, the teacher will have to choose the content to be taught through flipped classroom approach which is an issue of content knowledge (CK). The resources to be made available to the students at home in the form of videos will also be decided in correspondence with the content to be taught which involves the TK and TCK aspects of TPACK. In addition to this, the provision of in-class and outside the class activities by the teacher is also the part of flipped classroom approach which is related to PK and PCK.

4. Conclusion

To conclude, it can be said that considering the students' fear for the subject mathematics, it is time to rethink the pedagogical strategies for mathematics for the better learning of students and an adequate level of TPACK will enable the mathematics teacher to appropriately select and use the innovative pedagogical strategies like demonstration of concrete and virtual manipulatives, flipped classroom approach, dynamic mathematics software and animated content demonstration in the mathematics classroom.

Thus, the mathematics teacher should continuously strive towards the improvement in TPACK for the better learning of students. The TPACK level of teachers can be improved through deliberate efforts like training or workshops. Such practices can lead towards the development of their TPACK which will help them in appropriate selection, use of pedagogical strategies and technologies corresponding to particular content in the mathematics classroom.

References

Agudo, J. E., Sanchez, H., & Rico, M. (2010). *Playing games on the screen: Adapting mouse interaction at early ages*. Proceedings of the 10th IEEE International Conference on Advanced Learning Technologies (pp. 493–497), Sousse, Tunisia.

.Bouck, E. C., Satsangi, R., Doughty, T. T., & Courtney, W. T. (2014). Virtual and concrete manipulatives: A comparison of approaches for solving mathematics problems for students with autism spectrum disorder. *Journal of Autism and Developmental Disorders*, *44* (1), 180–193.

Chai, C. S., Koh, J. H. L., & Tsai, C. C. (2010). Facilitating Pre-service teachers'development of Technological Pedagogical and Content Knowledge (TPACK). *Educational Technology & Society*, *13(4)*, 63-73.

Chai, C.-S., Koh, J. H.-L., & Tsai, C. C. (2016). A Review of TechnologicalPedagogical Content Knowledge. *Educational Technology & Society*, *16*(2), 31–51.

David, L. (2017). Talking to E.T.? Why Math May Be the Best Language.Retrieved on December 2, 2017 from Live Science website, https://www.livescience.com/59448-making-contact-aliens-language-math.html

Lavicza, Z., &Papp-Varga, Z. (2010). Integrating GeoGebra into IWB-equipped teaching environments: preliminary results, *Technology, Pedagogy and Education*, *19*(2), 245–252.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2007). *Mathematics teaching today: Improving practice, improving student learning* (2nd Ed.). Reston,VA: Author.

Satsangi, R., & Miller, B. (2017). The case for adopting virtual manipulatives in mathematics education for students with disabilities. *Preventing School Failure: Alternative Education for Children and Youth, 61*(4), 303-310.

Straesser, R. (2001). Cabri-g´eom`etre: Does Dynamic Geometry Software (DGS) change geometry and its teaching and learning?*International Journal of Computers for Mathematical Learning*, *6*(3), 319–333.

Suh, J., & Moyer, P. S. (2007). Developing students' representation fluency using virtual and physical algebra balances. Journal of Computers in Mathematics and Science Teaching, *26*(2), 155–173.

Tatar, E., & Zengin, Y. (2016). Conceptual understanding of definite integral with GeoGebra. *Computers in the Schools, 33*(2), 120-132.

Zengin, Y., & Tatar, E. (2017). Integrating Dynamic Mathematics Software into Cooperative Learning Environments in Mathematics. *Educational Technology & Society, 20* (2), 74–88.

Zengin, Y. (2017). Investigating the Use of the Khan Academy and Mathematics Software with a Flipped Classroom Approach in Mathematics Teaching. *Educational Technology & Society, 20* (2), 89–100.

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